WEIGHT DISTRIBUTION OF THE HERMITIAN REED-MULLER CODES

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Abstract. We recall the construction of the Reed-Muller hermitian codes and some results about hermitians forms and exponential sums. With the help of these results, we give the weight distribution of the Reed-Muller hermitian codes and give some examples for which experimental computations have been made.

keywords

Hermitian Codes, Trace Codes, weight distribution.

Contents

1. INTRODUCTION

In this paper we study the weight distribution of Reed-Muller hermitian codes and some sub-codes of these codes. In all the paper q will be a power of a prime number p and $\mathbb{F}_q$ the field with q elements. We denote by W a vector space on $\mathbb{F}_{q^2}$ of dimension n and V the F_q-subspace subjacent to W. In fact, we can take $W = \mathbb{F}_{q^2}^n$ and $V = \mathbb{F}_q^{2n}$. In the second section, we recall the definitions and the properties of B_H and f_H respectively the hermitian form and the quadratic hermitian form on V associated to a sesquilinear form H on W for the involution $x \mapsto x^q$. We refer to [1] for a general presentation of these forms. The two main results of this section are proposition 1 and proposition 3, which uses results about exponential sums [3]. The third section contains the definition of the hermitian codes, of their trace codes and some of their properties. We also introduce an interesting sub-code of these codes and compute its

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length and its dimension. For an example ($q = r = 2$, $n = 2$) we give an explicite calculation of the quadratic hermitian forms on V and deduce from it the weight distribution.
In the two last sections we give the weight distribution of these codes, which was as far as we know an open problem. we have experimented our results in several cases, with the help of informatic programs. One can compare our results with the ones obtained in the quadratic case in [5].

2. HERMITIAN MATRIX AND HERMITIAN FORMS

Definition 1. *A square matrix $M = (m_{ij})$ with coefficients in $\mathbb{F}_{q^2}$ is said to be* hermitian *if M satisfies one of the following equivalent conditions :*

- $m_{ij} = (m_{ji})^q$, *for all i and j in $\{1, 2, ..., n\}$*
- $M^T = M^{(q)}$, *where $M^{(q)} = (m_{ij}^q)_{1 \leq i \leq n, 1 \leq j \leq n}$*

The set of all hermitian matrices will be noted $\Upsilon(n)$ and we have ([3])

$$\#\Upsilon(n) = q^{n^2}.$$

Definition 2. *([1]) Two hermitian matrix H and G are said to be equivalent if there exists a non singular matrix A with coefficients in $\mathbb{F}_{q^2}$ such that*

$$A^T H A^{(q)} = G.$$

The following result holds : any hermitian matrix of rank ρ and order n is equivalent to a diagonal matrix $D(\rho) = (a_{ij})$ where $a_{ij} = 0$, except if $i = j = 1, 2, ..., \rho$.

Proposition 1. *Let $N(n, \rho)$ be the number of hermitains matrix of order n and rank ρ, with coefficient in $\mathbb{F}_{q^2}$. Then we have*

$$1 + \sum_{\rho=1}^{n} N(n, \rho) = q^{n^2}$$

$$N(n, \rho) = \; q^{\rho(\rho-1)/2} \prod_{i=1}^{\rho} \frac{q^{2n-2(\rho-i)} - 1}{q^i - (-1)^i}$$

The first assertion is obvious and the proof of the second can be found in ([2], [4]).

Now we are going to recall some notions about quadratic hermitian forms on V. Let H be a sesquilinear form for the involution $x \mapsto x^q$ on W and $\Upsilon(W)$ the set of all these forms. The form H satisfies the following conditions :

- H is linear in its first variable
- $H(x,y) = H(y,x)^q$, for all (x,y) in $(\mathbb{F}_{q^2})^2$.

The *quadratic hermitian form* associated to H, is defined by

$$f_H(x) = H(\iota(x), \iota(x)), \text{ for all } x \in V$$

where $\iota : V \to W$ is the F_q-linear isomorphism defined by :

$$\iota(x_1, x_2, ..., x_{2n-1}, x_{2n}) = (x_1 + \alpha x_2, ..., x_{2n-1} + \alpha x_{2n})$$

where α is a primitive element of $\mathbb{F}_{q^2}$, i.e. $\mathbb{F}_{q^2} = \mathbb{F}_q(\alpha)$.

We shall denote by $QH(V)$ the set of the quadratic hermitian forms on V.

We now associate a quadratic form $f_H \in QH(V)$ to the $\mathbb{F}_q$-bilinear form B_H from $V \times V$ to $\mathbb{F}_q$ defined by :

$$B_H(x,y) = Tr_{F_{q^2}/F_q}(H(x,y))$$

The quadratic form f_H satifies the following relations, for all (x,y) in $V \times V$

- $f_H(x+y) = f_H(x) + f_H(y) + B_H(x,y)$
- $2f_H(x) = B_H(x,x)$

Then the bilinear form associated to f_H is $(1/2)B_H(x,y)$ if the caracteristic of the field is not 2.

The map $H \to f_H$ from $\Upsilon(W)$ to $Q(V)$, set of the quadratic forms on V, is injective ([3]). The image of this map is isomorphic to $QH(V)$([3]).

We also need the following definition.
The kernel of B_H is the set

$$V^0(B_H) = \{x \in V, B_H(x,y) = 0, \text{ for all } y \in V\}$$

The orthogonal supplement of $V^0(B_H)$ for the usual scalar product of the vector space V will be noted

$$V^1(B_H) = \{x \in V, x.y = 0, \text{ for all } y \in V^0(B_H)\}.$$

We have the following proposition.

Proposition 2. *1)There exists an endomorphism T_H of V such that for all (x, y) in $V \times V$*

$$B_H(x, y) = T_H(x).y.$$

2) Moreover, the endomorphism T_H satisfies :

- $\ker T_H = V^0(B_H)$
- $\mathrm{Im}\, T_H = V^1(B_H)$
- $\ker T_H \subset (f_H)^{-1}(0)$
- $V = V^0(B_H) \oplus V^1(B_H)$

If ρ is the rank of H, the dimension of the vector sub-space $V^1(B_H)$ is 2ρ. This number will be called the *rank* of f_H.

Remark. In the last sections we shall have to consider for a vector v the set $(T_H)^{-1}(v)$. This set is equal to $u+\ker T_H$, when exists a vector u such that $T_H(u) = v$ and then f_H is constant on this set. Let us prove this result. If u' belongs to $Ker(T_H)$ and u satisfies $T_H(u) = v$, then

$$f_H(u + u') = f_H(u) + f_H(u') + B_H(u', u)$$

But as $f_H(u') = 0$, according to proposition 2 and $B_H(u', u) = T_H(u').u = 0$, so

$$f_H(u + u') = f_H(u).$$

We shall also need to know the number of solutions in $(\mathbb{F}_q)^{2n}$of the equation

$$Tr_{\mathbb{F}_q/\mathbb{F}_r}(f_H(x) + v.x) = 0$$

where r is a positive integer, such that $\mathbb{F}_p \subset \mathbb{F}_r \subset \mathbb{F}_q$, v a fixed vector in V and $v.x$ the usual scalar product on V. Let us note $N_v(f_H)$ this number. An extension of a proposition of [3] gives us the following proposition, whose proof uses exponential sums.

Proposition 3. *1) If the vector v belongs to $V^1(B_H)$ two cases may occurs*

if the restriction of f_H on the set $(T_H)^{-1}(v)$ is equal to 0, then

$$N_v(f_H) = r^{-1}q^{2n} + r^{-1}(-1)^{\rho}q^{2n-\rho}(r-1)$$

if the restriction of f_H on the set $(T_H)^{-1}(v)$ is not equal to 0, then

$$N_v(f_H) = r^{-1}q^{2n} + r^{-1}(-1)^{\rho}q^{2n-\rho}$$

2) If the vector v does not belong to $V^1(B_H)$ then

$$N_v(f_H) = r^{-1}q^{2n}.$$

3. HERMITIAN CODES

We are now able to recall the construction and the properties of the Reed-Muller hermitian codes. The code C is the image of the injective map

$$\begin{array}{ccc} c : QH(V) \times V & \rightarrow & (F_q)^{\#V} \\ (f_H, v) & \mapsto & [f_H(x) + x.v]_{x \in V} \end{array}$$

In other words, the codeword associated to the couple (f_H, v) is the N-uple

$$(f_H(x_1) + x_1.v, ..., f_H(x_N) + x_N.v)$$

where N is equal to $\#V = q^{2N}$.

More generally we are going to consider the *trace codes* of these codes ([3]). If r is a positive integer such that q is one of its power (hence we have $\mathbb{F}_p \subset \mathbb{F}_r \subset \mathbb{F}_q$, where p is a prime integer) , the trace code Γ of C is defined as the image of the map

$$\begin{array}{ccc} \gamma : QH(V) \times V & \rightarrow & (F_r)^{\#V} \\ (f_H, v) & \mapsto & [Tr_{\mathbb{F}_q/\mathbb{F}_r}(f_H(x) + x.v)]_{x \in V} \end{array}$$

From proposition 5 of [3] we deduce :

Proposition 4. *The parameters of the code* Γ *are*

$$[N_\Gamma, K_\Gamma, D_\Gamma] = [q^{2n}, n(n+2)\log_r(q), r^{-1}q^{2n-1}(qr - q - 1)]$$

We shall also consider the subcode Γ_0 of Γ, which is the image of the injective map

$$\begin{array}{ccc} \gamma_0 : QH(V) \times V & \rightarrow & (F_r)^{\#V} \\ (f_H, v) & \mapsto & [Tr_{\mathbb{F}_q/\mathbb{F}_r}(f_H(x))]_{x \in V} \end{array}$$

The length of this code is q^{2n}, as in the general case and the dimension is clearly $n^2 \log_r(q)$. The minimal distance will be computed in the following section.
Of course this last definition includes the case $q = r$ and in this case we shall note C_0 the code obtained.

Example: Let us give an example of such a code. We take $q = r = 2$ and $n = 2$. We are first going to describe the set $QH(V)$ in this case. We take W equal to $(\mathbb{F}_4)^2$ as mentionned in the introduction and V equal to $(\mathbb{F}_2)^4$. We have also

$$\mathbb{F}_4 = \mathbb{F}_2(\alpha)$$

with $\alpha \in \mathbb{F}_4$ satisfying $\alpha^2 + \alpha + 1 = 0$. (The polynomial $X^2 + X + 1$ is irreductible on $\mathbb{F}_2[X]$).

If H is a sesquilinear form for the involution $x \mapsto x^2$ on W, there exists an hermitian matrix M_H such that

$$H(x,y) = (x_1, x_2) M_H \begin{pmatrix} y_1^2 \\ y_2^2 \end{pmatrix}$$

where:

- $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are two vectors of $(\mathbb{F}_4)^2$
- $M_H = \begin{pmatrix} u & \xi^2 \\ \xi & v \end{pmatrix}$, with $(u, v) \in \mathbb{F}_2 \times \mathbb{F}_2$ and $\xi \in \mathbb{F}_4$.

Then we have

$$H(x,y) = ux_1^3 + \ vx_2^3 + \xi x_2 x_1^2 + \xi^2 x_1 x_2^2$$

By definition f_H is equal to $H(\iota(x), \iota(x))$, for $x = (x_1, x_2, x_3, x_4)$ in V and with, in this case

$$\iota(x_1, x_2, x_3, x_4) = (x_1 + \alpha x_2, x_3 + \alpha x_4)$$

If we put $\xi = \mu + \nu\alpha$ we obtain

$$f_H(x) = u(x_1 + x_2 - x_1x_2) + v(x_3 + x_4 - x_3x_4) + \mu(x_1x_4 - x_2x_3) + \nu(x_1x_3 + x_1x_4 + x_2x_4)$$

with each number u, v, μ and ν in $\mathbb{F}_2$. The length of the subcode C_0 is 2^4 and is equal to the cardinal of V . The dimension of C_0 is 4 and is equal to $\dim(\Upsilon(W))$. The following list of the codewords is easily computed with the help of the previous formula.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 1 0 1 0 1 1 0 0 0 1 1
0 0 0 0 0 0 1 1 0 1 0 1 0 1 1 0	0 0 0 0 0 1 1 0 0 0 1 1 0 1 0 1
0 1 1 1 0 1 1 1 0 1 1 1 0 1 1 1	0 1 1 1 0 0 1 0 0 0 0 1 0 1 0 0
0 1 1 1 0 1 0 0 0 0 1 0 0 0 0 1	0 1 1 1 0 0 0 1 0 1 0 0 0 0 1 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1	0 0 0 0 1 0 1 0 1 0 0 1 1 1 0 0
0 0 0 0 1 1 0 0 1 0 1 0 1 0 0 1	0 0 0 0 1 0 0 1 1 1 0 0 1 0 1 0
0 1 1 1 1 0 0 0 1 0 0 0 1 0 0 0	0 1 1 1 1 1 0 1 1 1 1 0 1 0 1 1
0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0	0 1 1 1 1 1 1 1 0 1 0 1 1 1 1 0 1

So we note that the weights of the codewords are $0, 6$ and 12 with one codeword of weight 0, 10 codewords of weight 6 and 5 codewords of weight 12.

4. WEIGHT DISTRIBUTION OF Γ_0

4.1. Weight Distribution of Γ_0. This subcode is, as recalled before, the image of the map

$$\begin{array}{ccc} \gamma_0 : QH(V) & \rightarrow & (F_r)^{\#V} \\ f_H & \mapsto & [Tr_{F_q/F_r}(f_H(x))]_{x\in V} \end{array}$$

The weight $w(\gamma_0(f_H))$ of the codeword constructed from f_H is given by the following relation

$$w(\gamma_0(f_H)) = q^{2n} - \#\{x \in V \mid Tr_{\mathbb{F}_{q/\mathbb{F}_r}}(f_H(x)) = 0\}.$$

Let's put with the notations of the second section

$$N_v(f_H) = \#\{x \in V \mid Tr_{F_q/F_r}(f_H(x) + x.v) = 0\}$$

for each vector $v \in V$.
For the subcode Γ_0, v is equal to zero and belongs to $V^1(B_H)$ for all f_H in $QH(V)$. Then, as in this case $(T_H)^{-1}(v)$ is simply equal to $\ker T_H$ which is included in $(f_H)^{-1}(0)$, we have according to proposition 3 and for all f_H in $QH(V)$

$$w(\gamma_0(f_H)) = q^{2n} - r^{-1}q^{2n} - \ r^{-1}(-1)^\rho q^{2n-\rho}(r-1).$$

The weight $w(\gamma_0(f_H))$ depends only of the rank ρ of H. So we put $w(\gamma_0(f_H)) = w_\rho$. The map

$$\rho \longmapsto w_\rho = q^{2n} - r^{-1}q^{2n} - \ r^{-1}(-1)^\rho q^{2n-\rho}(r-1)$$

is clearly injective. Then the weights of the codewords are the number w_ρ with $0 \leq \rho \leq n$. The number of codewords of a given weight w_ρ is simply the number of hermitian matrix of rank ρ given in proposition 1. We have

Theorem 5. *For the code Γ_0 we have the following assertions.*

- *The weights of the codewords are the numbers*

$$w_\rho = q^{2n} - r^{-1}q^{2n} - \ r^{-1}(-1)^\rho q^{2n-\rho}(r-1)$$

  for $\rho = 0, 1, ..., n$.

- *The number A_ρ of codewords of weight w_ρ is equal to*

$$A_\rho = N(n,\rho) = \ q^{\rho(\rho-1)/2}\prod_{i=1}^{\rho}\frac{q^{2n-2(\rho-i)}-1}{q^i-(-1)^i}$$

We have experimented this result with the help of a program which computes the words of the code Γ_0 and their weight, in the following cases :

- $q = r = 2\ ;\ n = 2$.

  This code contains $2^4 = 16$ words and has been studied at the end of the third section. The distribution of the weights is :

ρ	w_ρ	A_ρ
0	0	1
1	12	10
2	6	5

  We recover the result explicitly found in section three.

- $q = r = 2\ ;\ n = 3$.

  This code contains $2^9 = 512$ words. The distribution of the weight is :

ρ	w_ρ	A_ρ
0	0	1
1	48	21
2	24	210
3	36	280

- $q = r = 3\ ;\ n = 2$.

  This code contains $3^4 = 81$ words. The distribution of the weights is

ρ	w_ρ	A_ρ
0	0	1
1	72	20
2	48	60

4.2. Minimal distance of the code Γ_0. This distance is the smallest weight strictly positive of all the codeword of Γ_0. In our particular case, we have to find the following minimum

$$\min_{1\leq\rho\leq n}\{w_\rho = q^{2n} - r^{-1}q^{2n} - \ r^{-1}(-1)^\rho q^{2n-\rho}(r-1)\}$$

It is easy to see that this minimum is taken for an odd value of ρ, and precisely for the value which maximise $q^{2n-\rho}(r-1)$, that is $\rho = 2$. This gives the theorem

Theorem 6. *The parameters $[N_0, K_0, D_0]$ of the code Γ_0 are*

$$[N_0, K_0, D_0] = [q^{2n},\ n^2\log_r(q),\ r^{-1}(r-1)q^{2n-2}(q^2-1)]$$

Remark. Comparing with proposition 3 and with the same notations, we have

$$D_0 = D_\Gamma + r^{-1}q^{2n-2}(q-r+1)]$$

For example an experimental computation shows that in the case $q = r = 2,\ n = 2$ the minimal distance of Γ (here C, with the previous notations) is equal to 4. Theorem 5, and the computation above shows that the minimal distance D_0 is equal to 6.

5. WEIGHT DISTRIBUTION OF Γ

This Hermitian Reed-Muller code is the image of the map γ

$$\begin{array}{ccc} QH(V) \times V & \rightarrow & (F_r)^{\#V} \\ (f_H, v) & \mapsto & [Tr_{\mathbb{F}_{q/\mathbb{F}_r}}(f_H(x) + v.x)]_{x \in V} \end{array}$$

Let us note $\gamma(f_H, v)$ a codeword and $w(\gamma(f_H, v))$ its weight. As in the preceding section this weight is

$$w(\gamma(f_H, v)) = q^{2n} - \#\{x \in V \mid Tr_{\mathbb{F}_{q/\mathbb{F}_r}}(f_H(x) + x.v) = 0\} = q^{2n} - N_v(f_H)$$

For $\gamma(f_H, v)$ three cases, given by proposition 3, may occurs.

- If v belongs to $V^1(B_H)$ and $f_{H|(T_H)^{-1}(v)} = 0$ ($f_{H\,|\,(T_H)^{-1}(v)}$ is the restriction of f_H to $(T_H)^{-1}(v)$), then

$$w(\gamma(f_H, v)) = q^{2n} - r^{-1}q^{2n} + r^{-1}(-1)^{\rho+1}q^{2n-\rho}(r-1) = W_\rho.$$

- If v belongs to $V^1(B_H)$ and $f_{H|(T_H)^{-1}(v)} \neq 0$, then

$$w(\gamma(f_H, v)) = q^{2n} - r^{-1}q^{2n} + r^{-1}(-1)^{\rho}q^{2n-\rho} = W'_\rho.$$

- If v does not belong to $V^1(B_H)$, then

$$w(\gamma(f_H, v)) = q^{2n} - r^{-1}q^{2n}.$$

We have first the obvious lemma.

Lemma 7. *The number W_ρ for $0 \leq \rho \leq n$, W'_ρ for $1 \leq \rho \leq n$ and $q^{2n} - r^{-1}q^{2n}$ are mutually distincts.*

Then in order to know the distribution of weights, we only need to count the number of couples (f_H, v) for each rank ρ satifying the first case, because we already know the cardinal of $QH(V) \times V$ and of $V^1(B_H)$. We have

Lemma 8. *The number of couples $(f_H, v) \in QH(V) \times V$, with f_H of rank ρ, such that $v \in V^1(B_H)$ and $f_{H\,|\,(T_H)^{-1}(v)} = 0$ is equal to the product $S_\rho N(n,\rho)$, where :*

$$S_\rho = q^{2\rho-1} + (-1)^\rho q^{\rho-1}(q-1).$$

Proof. Let S_ρ be the cardinal of the set $\{\ v \in V^1(B_H)\ |\ \ f_{H\,|\,(T_H)^{-1}(v)} = 0\}$. With the help of the second section, we can say that $(f_H)^{-1}(0)$ is the disjoint union of the sets $(T_H)^{-1}(v)$ for $v \in V^1(B_H)$ such that $f_{H\,|\,(T_H)^{-1}(v)} = 0$. As $\#(T_H)^{-1}(v) = \#\ker T = q^{2n-2\rho}$, we have

$$\#(f_H)^{-1}(0) = q^{2n-2\rho} S_\rho$$

The proposition 3 gives the cardinal of $(f_H)^{-1}(0)$

$$\#(f_H)^{-1}(0) = q^{-1}\left(q^{2n} + (-1)^\rho q^{2n-\rho}(q-1)\right).$$

$$S_\rho = q^{2\rho-1} + (-1)^\rho q^{\rho-1}(q-1). \blacksquare$$

For a couple $(f_H, v) \in QH(V) \times V$, such that $v \notin V^1(B_H)$, the weight of the word $\gamma(f_H, v)$ does not depend of the rank ρ and is equal to

$$w(\gamma(f_H, v)) = q^{2n} - r^{-1}q^{2n}.$$

Recalling that

$$\#(V \backslash V^1(B_H)) = q^{2n} - q^{2\rho}$$

we obtain that Γ contains

$$\sum_{\rho=0}^{n-1} (q^{2n} - q^{2\rho}) N(n,\rho)$$

words of weight $q^{2n} - r^{-1}q^{2n}$. We can summarise these calculations in the following theorem.

Theorem 9. *For the code Γ we have:*

- *the weights of the codewords are the numbers*

$$W_\rho = q^{2n} - r^{-1}q^{2n} + \ r^{-1}(-1)^{\rho+1}q^{2n-\rho}(r-1),\ \text{for } \rho = 0, 1, ..., n$$

$$W_\rho^{'} = q^{2n} - r^{-1}q^{2n} + \ r^{-1}(-1)^{\rho+1}q^{2n-\rho},\ \text{for } \rho = 1, ..., n$$

$$W = q^{2n} - r^{-1}q^{2n}.$$

- *If $A_\rho(w)$ represents the number of codeword of weight w in Γ, then*

$$A(W_\rho) = S_\rho N(n,\rho) \quad ; A(W_\rho^{'}) = (q^{2\rho} - S_\rho)N(n,\rho)$$

$$A(W) = \sum_{\rho=0}^{n-1}(q^{2n} - q^{2\rho})N(n,\rho)$$

where

$$S_\rho = q^{2\rho-1} + (-1)^\rho q^{\rho-1}(q-1)$$

$$N(n,\rho) = \; q^{\rho(\rho-1)/2}\prod_{i=1}^{\rho}\frac{q^{2n-2(\rho-i)}-1}{q^i-(-1)^i}$$

As in the preceding case, we have experimented these results in the following cases :

- $q = r = 2\ ;\ n = 2$.

  This code Γ contains $2^8 = 256$ words. In this case we have :

$$W_0 = 0\ ;\ W_1 = 12\ ;\ W_2 = 6\ ; W_1^{'} = 4\ ;\ W_2^{'} = 10\ ;\ W = q^{2n} - r^{-1}q^{2n} = 8$$

  The distribution of the weights is given in table 1.

w	$A_\rho(w)$
0	1
4	15
6	100
8	75
10	60
12	5

table1

w	$A_\rho(w)$
0	1
16	63
24	2100
28	10080
32	11403
36	7840
40	1260
48	21

table2

- $q = r = 2\ ;\ n = 3$.

  This code contains $2^{15} = 32768$ words. The distribution of the weights calculated with theorem 9 and experimentally confirmed is given in table 2.

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